Audio power amplifier design — 6

More on negative feedback and non-linearity distortion — a continuation of part 5

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Part 5 (December issue) discussed the theory of non-linearity distortion in an ideal feedback amplifier having a parabolic forward transfer characteristic. Attention is now turned to distortion in circuits using ordinary junction transistors*, having exponential transfer characteristics. The concept of "inverse distortion" is introduced, leading to a useful distortion theorem.

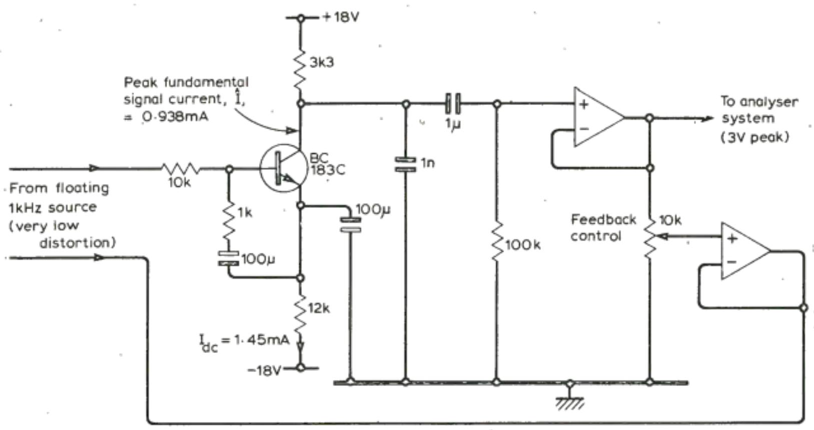

Fig. 1. *Circuit used for distortion measurements.*

THE CIRCUIT USED for obtaining the experimental results presented below is shown in Fig. 1 and is the same as for the f.e.t. tests in Part 5, except for two small modifications. The 1nF capacitor was found necessary to prevent high-frequency oscillation when full feedback was applied, and the resistive attenuator in the base circuit was added to reduce loop gain, for convenience, to a similar range of values to that applying to the f.e.t. version of the circuit. The measured current gain (β_{dc} or h_{FE} of the transistor used was 580 at, an I_b of 1μA.

Throughout the measurements the fundamental output voltage was kept constant at three volts peak, corresponding to a ratio of peak signal current to direct working current of 0.647 – the same conditions as for the f.e.t. tests in Part 5. The results are shown by the full-line curves in Fig. 2, and exhibit some fascinating features when compared with the earlier f.e.t. results. A great deal of thought, both of a formally analytical and also of a more intuitive type, has been devoted to trying to understand these features, and considerable enlightenment has resulted.

A junction transistor has the great virtue, at sufficiently low values of collector current, that it follows in practice, with high accuracy, the relationship

$$I_c = I_o \exp\frac{qV_{be}}{kT} \qquad (1)$$

where I_c is collector current, V_{be} base-to-emitter voltage, and the other symbols are constants. (The tendency always to regard junction transistors as current-operated devices, and the current gain as the basic parameter for design purposes, should be most strongly discouraged, in my opinion.)

A practical junction transistor would be expected to follow the above law much more closely than an f.e.t. would be expected to follow a parabolic law, so that there seemed good reason for thinking that the curious wiggles in the Fig. 2 curves might be theoretically explicable on the basis of equation 1.

Determining transfer characteristic

For analysis the circuit may be simplified to that shown in Fig. 3, in which the transistor is assumed to follow equation 1. It may be shown that the incremental signal input and output voltages of the circuit are related by

$$v_{out} = -R_L I_{dc}\left[\exp\frac{qv_{in}}{kT} \times \exp\frac{q\beta v_{out}}{kT} - 1\right] \qquad (2)$$

where q is the electronic charge (1.60×10^{-19} coulomb) k Boltzmann's constant (1.38×10^{-23} joules/deg C) and T absolute temperature. To be able to calculate the harmonics in v_{out} when v_{in} in equation 2 is put equal to $\hat{V}_{in}\sin\omega t$, the relation must be expressed in the form of a power series:

$$v_{out} = a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4 + \ldots \qquad (3)$$

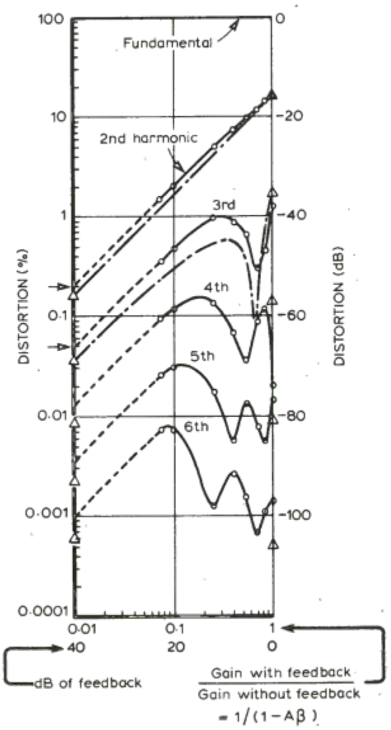

Fig. 2. *Measured and calculated results for the Fig. 1 circuit.*

*Sometimes called bipolar junction transistors or b.j.t. because their operation involves both polarities of charge carrier. The usual type of f.e.t. is also a junction device, but it is unipolar because only one polarity of charge carrier is involved.

The values of the coefficients a_1, a_2, a_3 etc may be found by using Maclaurin's theorem, which says

$$a_1 = \left[\frac{dv_{out}}{dv_{in}}\right]_{vin=0}$$

$$a_2 = \frac{1}{2!}\left[\frac{d^2v_{out}}{dv_{in}^2}\right]_{vin=0}$$

$$a_3 = \frac{1}{3!}\left[\frac{d^3v_{out}}{dv_{in}^3}\right]_{vin=0}$$

By successively differentiating equation 2 and putting $v_{in}=0$ in the resultant expressions, the coefficients may thus be determined. Unfortunately the algebra rapidly becomes cumbersome, and being no mathematician, I gave up after determining the first three coefficients, which are

$$a_1 = \frac{A}{1-A\beta} \quad (4)$$

$$a_2 = \frac{1}{2!}\frac{q}{kT}\frac{A}{(1-A\beta)^3} \quad (5)$$

$$a_3 = \frac{1}{3!}\left(\frac{q}{kT}\right)^2 A\left[\frac{1}{1-A\beta} - \frac{3|A\beta|}{(1-A\beta)^2} + \frac{3|A\beta|^2}{(1-A\beta)^3} - \frac{|A\beta|^3+3|A\beta|}{(1-A\beta)^4} + \frac{3|A\beta|^2}{(1-A\beta)^5}\right] \quad (6)$$

In these equations β is positive and $A=-g_mR_L$, where g_m is the transistor mutual conductance when $v_{in}=v_{out}=0$ and the collector current is I_{dc}.

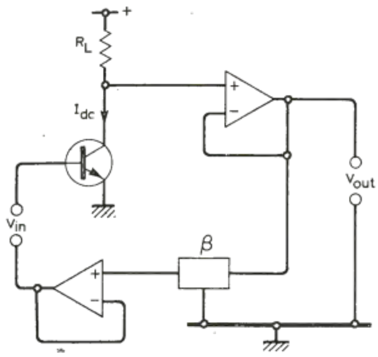

Fig. 3. *Simplified version of Fig. 1 with d.c. bias arrangements omitted.*

Determining the harmonics

Knowing the value of v_{in} ($=\hat{V}_{in}\sin\omega t$), as a function of the amount of feedback in use, for the output level of 3V peak adopted, the output harmonic magnitudes may be calculated from equation 3 on the assumption that only the square-law term is responsible for the second harmonic and only the cubic term for the third harmonic. Because the output level is large, this simplifying assumption leads to appreciable, though not unduly gross, errors, and for better accuracy the production of some second harmonic due to the presence of a fourth-power term needs to be taken into account, etc. A fairly high output level was adopted in the experiments to make the high-order harmonics sufficiently large for straightforward measurement, i.e. well over 0.0001%.)

The calculated second and third-harmonic curves are shown chain-dotted in Fig. 2, and lie somewhat below the measured curves because of the above simplifying assumption. The reasons for other detailed differences will become apparent later on.

In view of the + and − signs in front of the terms in equation 6, and on the supposition that the expressions for a_4, a_5 etc. will contain even more terms of both signs, one can at least say that it is hardly surprising that the measured curves for the higher-order harmonics in Fig. 2 are of a more complex type.

Alternative approach

The method of analysis presented above basically involves determining the transfer characteristic for the complete feedback amplifier, and then calculating the harmonic magnitudes when a sine-wave input is handled via this transfer characteristic. The shape of the overall transfer characteristic changes as the amount of feedback is altered, resulting in the observed variation in the magnitudes of the various harmonics. It should be emphasized that no intermodulation concept is involved in this approach when the input to the complete circuit is a single sine-wave signal.

An alternative approach, which is very helpful in providing further insight, involves thinking simply in terms of the invariant transfer characteristic of the forward amplifying device. Intermodulation effects then *do* have to be taken into account, for the forward amplifying device receives inputs from both the sine-wave input signal and also via the β-network from the amplifier output, the last-mentioned contribution containing harmonics which intermodulate with the fundamental and with each other.

In particular, the second harmonic and fundamental intermodulate to produce a component at third-harmonic frequency, and careful consideration of the waveform polarities involved shows that this third-harmonic component is in antiphase with that produced by straightforward third-harmonic distortion of the fundamental. The null in the curve for total third-harmonic distortion thus occurs when the amount of feedback is such as to make these oppositely-phased third-harmonic components of equal magnitude. The fact that in the measured third-harmonic curve, a minimum rather than a perfect null is observed, is believed to be because slight phase errors in the experimental circuit prevented the two third-harmonic components from being exactly in antiphase.

A further intermodulation effect is that the fundamental and third harmonic intermodulate to produce a second-harmonic component which, though of considerably smaller magnitude than that produced by straightforward second-harmonic distortion of the fundamental, nevertheless slightly modifies the shape of the second-harmonic curve.

The percentage of second harmonic generated within the transistor, at constant output, is proportional to the third-harmonic voltage fed back into the base circuit, but the percentage output distortion is reduced (1−Aβ) times relative to this by negative feedback. For working conditions to the left of the null in the third-harmonic curve, this intermodulation-generated second harmonic is in the same phase as that produced by straightforward second-harmonic distortion. Its magnitude at the amplifier output, with enough feedback to bring the working point onto the approximately constant-slope part of the third-harmonic curve, is such as to lift the position of the second-harmonic curve by a constant distance, and the calculated spacing is of the order shown.

It is instructive to compare the Fig. 2 curves with those of Fig. 7 in Part 5. There are two basic differences (a) the f.e.t. curves show no nulls or minima; and (b) the measured f.e.t. second-harmonic curve does not exhibit the departure from linearity evident in the Fig. 2 curve. The reason for (a) above is believed to be that, for the f.e.t. specimen used, the harmonic-distortion-generated third-harmonic component was in phase, rather than in antiphase, with the component generated by intermodulation. The high-order terms in the transfer characteristic for an f.e.t., unlike those for a junction transistor, seem to vary from one specimen to another – the one used for the Fig. 7 (Part 5) results had been selected for low third harmonic. It may well be that some other speciments would give curves with nulls, but this has not been investigated.

The reason for difference (b) above is simply that the signal level was too low to make the effect noticeable. Though the f.e.t. and the junction transistor were both worked at the same ratio of peak signal to direct working current, the f.e.t., because of its different type of transfer characteristic, gave less second-harmonic distortion in the absence of feed-back – see equations 16 and 17 in Part 5. On turning up the signal level in the f.e.t. circuit for 4V rather than 3V peak output, an appreciable departure of the second-harmonic curve from linearity was observed.

High-feedback theory

It is a characteristic of the Fig. 2 curves that all their complex features disappear when enough feedback is applied, and this fact suggests that maybe the high-feedback parts of the curves at the left could be calculated in a manner devoid of the above complications. This indeed turns out to be the case, and it is thought that appreciation of this fact is of considerable engineering value, for in the majority of practical applications one is really only interested in the performance with plenty of feedback applied.

Any amplifier, without feedback, can in principle be made to give a perfectly sinusoidal output voltage, at a specified level, by feeding an appropriately distorted waveform to its input. With negative feedback applied, this same totally undistorted output voltage can be maintained if V_{in} (Fig. 4) is arranged to contain the necessary distorted error voltage, as above, plus some extra fundamental to cancel the fundamental being injected negatively into the input circuit via the β-network. (With undistorted output, the feedback voltage is, of course, also perfectly sinusoidal.) Thus, as β is increased, V_{in} has to supply a constant-amplitude harmonic spectrum plus an increased amount of fundamental. The magnitude of the required fundamental input, for the specified constant output voltage V_{out}, is given by the usual feedback formula.

$$\frac{V_{out}}{V_{in}}=\frac{A}{1-A\beta}$$

which for the present purpose is more conveniently arranged as

$$V_{in}=V_{out}\frac{1-A\beta}{A}$$

Since the harmonic part of the input is quite constant, the percentage input distortion is inversely proportional to the amount of fundamental input voltage, i.e. it is proportional at $1/(1-A\beta)$, and this applies at every harmonic frequency. It also applies whether the amount of feedback is large or small.

It is thus seen that the distortion situation for a feedback amplifier is really very much simpler when viewed on this basis of percentage input distortion for a pure output, than when considered on the more usual basis of the output distortion for a pure sinusoidal input. At this point the reader may well object that, while it may indeed be easier to consider the feedback mechanism on this basis, the concept is artificial and not related to the way amplifiers are used in practice. The utility of the approach, however, lies in the fact that, *provided there is plenty of feedback*, the distortions become practically identical whether expressed on a distorted-input/pure-output basis, or on the usual distorted-output/pure-input basis. Thus if the percentage distortion with no feedback is calculated on a pure-output/distorted input basis – which turns out to be relatively easy – then the distortion with plenty of feedback applied, expressed in the customary manner, is equal to the just-mentioned no-feedback percentage divided by $(1-A\beta)$, the output level being kept constant. This applies both to total harmonic distortion and also to all individual harmonics of practical significance, provided only that the amount of feedback is sufficiently large. For the working conditions relevant to Fig. 2, or Fig. 7 of Part 5, it is evident that 20 to 26dB of feedback would be "sufficiently large."

It is now necessary to justify the statement that the distortion with plenty of feedback is practically the same whether expressed on a distorted-input/pure-output basis, or on a distorted-output/pure-input basis. With reference to Fig. 4, consider the state of affairs when V_{in} is of pure sine waveform, suitably adjusted in magnitude to maintain a constant output voltage no matter how much feedback there is. With no feedback, V' will be equal to V_{in} and will be sinusoidal, V_{out} being highly distorted. As the amount of feedback is increased, V_{out} becomes more and more nearly sinusoidal, which requires that the V' waveform must approximate more and more closely to that specific highly-distorted waveform, characteristic of the particular forward amplifier, which will make it deliver a perfectly sinusoidal output. The whole of the distortion in V' – call it V_{dist} – is supplied from the β-network, since V_{in} is pure. When the amount of feedback is large, the fundamental output from the β-network, injected into the input circuit, is very nearly equal in magnitude to V_{in}. Hence the percentage distortion in the output from the β-network, and therefore also in the amplifier output voltage, which feeds the β-network, is very nearly $(V_{dist}/V_{in})\times 100\%$. If now, with this large amount of feedback applied, a slight harmonic content is introduced into the V_{in} waveform so as to make the *output* perfectly sinusoidal, neither the magnitude of V_{in}, nor the harmonic content of the V' waveform, will change by more than a tiny amount, so that the distortion will still be given quite closely by $(V_{dist}/V_{in})\times 100\%$. Thus the larger the amount of feedback, the more nearly does the percentage output distortion for pure input become equal to the percentage input distortion for pure output.

Another argument to support the statement that the percentage distortions, with a large amount of feedback applied, are virtually the same when expressed on either basis mentioned above, is as follows. Referring to Fig. 4 again, suppose V_{in} contains the necessary harmonics to make V_{out} perfectly sinusoidal. Now, with these input harmonics still present, imagine that we add a further set of input harmonics, each of equal magnitude to, and in antiphase with, the corresponding harmonic already there. The result will be to cancel all the input harmonics, but introduce harmonics into V_{out}. If the harmonics thus introduced into V_{out} are simply the result of the faithful amplification of the additional set of

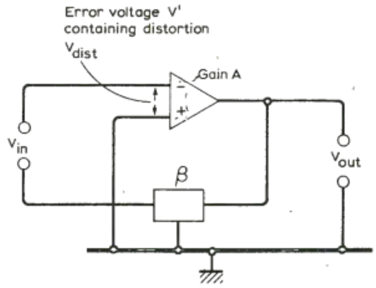

Fig. 4 *Basic feedback amplifier configuration.*

harmonics fed in, then it follows that the percentage distortions must be the same whether considered on a distortionless input or a distortionless output basis. Whether this is nearly enough the case for practical purposes depends on how low is the intermodulation distortion introduced by the complete feedback amplifier when fed with these small-amplitude additional input harmonics in the presence of a large fundamental input, and clearly the more feedback there is, the less significant will be any "false harmonics" introduced by intermodulation – intermodulation between the fundamental and the second harmonic might introduce some third harmonic, for example.

Thus, once again, the conclusion is reached that, *provided there is enough feedback*, harmonic-distortion percentages will be very nearly the same whether expressed on the normal pure-input basis, or on the inverse basis of input distortion for pure output.

A distortion theorem

The ideas discussed above may be formulated as a distortion theorem, applicable to total and to individual-harmonic distortion:

"The percentage harmonic distortion in the output of an amplifier having a large amount of feedback and a sine-wave input, is very nearly equal to the percentage input distortion for distortionless output without feedback, divided by $(1-A\beta)$."

The usefulness of this theorem is dependent on knowing the input distortion required to give distortionless output without feedback, for common amplifying devices and circuits, but fortunately the theory required is relatively simple. Such distortion can be termed "inverse distortion."

Junction transistor inverse-distortion theory

A simple single-ended junction-transistor stage will be considered first, the transistor being assumed to follow equation 1. When V_{be} is such as to cause I_c to vary sinusoidally,

$$I_{dc}+\hat{I}_c\sin\omega t=I_0\exp\frac{qV_{be}}{kT} \qquad (7)$$

in which V_{be} has the appropriate special waveform which it is desired to find. When $\hat{I}_c\sin\omega t$ passes instantaneously through zero

$$I_{dc}=I_o\exp\frac{qV_{dc}}{kT} \qquad (8)$$

where V_{dc} is the value of V_{be} required to establish the mean collector current I_{dc} in the absence of a signal input. Equation 7 may be written

$$\log_e\left[\frac{I_{dc}+\hat{I}_c\sin\omega t}{I_o}\right]=\frac{qV_{be}}{kT}$$

from which may be derived

$$V_{be}=\frac{kT}{q}\left[\log_e\frac{I_{dc}}{I_o}+\log_e(1+\frac{\hat{I}_c}{I_{dc}}\sin\omega t)\right] \quad (9)$$

But from equation 8,

$$\log_e\frac{I_{dc}}{I_o}=\frac{qV_{dc}}{kT},$$

so that equation 9 becomes

$$V_{be}=V_{dc}+\frac{kT}{q}\log_e(1+\frac{\hat{I}_c}{I_{dc}}\sin\omega t)$$

We now use the fact that

$$\log_e(1+x)=x-x^2/2+x^3/3-x^4/4+\ldots$$

which leads to

$$V_{be}=V_{dc}+\frac{kT}{q}\left[\frac{\hat{I}}{I_{dc}}\sin\omega t-\tfrac{1}{2}\left(\frac{\hat{I}}{I_{dc}}\right)^2\sin^2\omega t\right.$$
$$\left.+\tfrac{1}{3}\left(\frac{\hat{I}}{I_{dc}}\right)^3\sin^3\omega t-\tfrac{1}{4}\left(\frac{\hat{I}}{I_{dc}}\right)^4\sin^4\omega t+\right] \quad (10)$$

On the assumption that $\hat{I}/I_{dc}$ is not so large that, for example, the second harmonic generated by the $\sin^4\omega t$ term is large enough to cause serious error, equation 10 yields harmonic percentages as given in the middle column of Table 1. Since $g_m=qI_{dc}/kT$ and $\hat{I}=g_m\hat{V}_{in}$, we may replace $\hat{I}/I_{dc}$ by $q\hat{V}_{in}/kT$. At 290 K, which is approximately relevant to low-level stages at least, kT/q is 25mV. These facts enable the results in the right-hand column of Table 1 to be calculated.

Table 1. **Theoretical input distortion percentages for pure sinusoidal output from ideal junction transistor without feedback.**

Harmonic number	Distortion %	Distortion (%), alternative formulae for 290K (V_{in} in mV)
2	$25(\hat{I}/I_{dc})$	$\hat{V}_{in}$
3	$8.33(\hat{I}/I_{dc})^2$	$1.33\times10^{-2}\hat{V}_{in}^2$
4	$3.13(\hat{I}/I_{dc})^3$	$2.00\times10^{-4}\hat{V}_{in}^3$
5	$1.25(\hat{I}/I_{dc})^4$	$3.20\times10^{-6}\hat{V}_{in}^4$
6	$0.521(\hat{I}/I_{dc})^5$	$5.33\times10^{-8}\hat{V}_{in}^5$

Comparison with "normal" distortion

It is instructive to compare the Table 1 results with those giving the *output* distortion for an ideal sine-wave-driven junction transistor without feedback. Referring to equation 1, put $V_{be}=V_{dc}+\hat{V}_{in}\sin\omega t$, where V_{dc} is a direct bias voltage. This leads to

$$\frac{i_c}{I_{dc}}=\exp\frac{q\hat{V}_{in}\sin\omega t}{kT}-1 \qquad (11)$$

where i_c is the instantaneous signal component of collector current and I_{dc} the collector current when $\hat{V}_{in}\sin\omega t=0$. This time the required matematical fact is that

$$\exp x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots$$

which gives

$$\frac{i_c}{I_{dc}}=\frac{q}{kT}\hat{V}_{in}\sin\omega t+\frac{1}{2}\left(\frac{q}{kT}\right)^2\hat{V}_{in}{}^2\sin^2\omega t$$
$$+\frac{1}{6}\left(\frac{q}{kT}\right)^3\hat{V}_{in}{}^3\sin^3\omega t$$
$$+\frac{1}{24}\left(\frac{q}{kT}\right)^4\hat{V}_{in}{}^4\sin^4\omega t+\ldots \qquad (12)$$

The harmonic percentages may then be evaluated on the same basis as for Table 1, as functions of both $\hat{V}_{in}$ and $\hat{I}/I_{dc}$, since

$$\hat{V}_{in}=\frac{\hat{I}}{I_{dc}}\times\frac{kT}{q}$$

Substituting this in equation 12 gives

$$\frac{i_c}{I_{dc}}=\frac{\hat{I}}{I_{dc}}\sin\omega t+\frac{1}{2}\left(\frac{\hat{I}}{I_{dc}}\right)^2\sin^2\omega t+$$
$$\frac{1}{6}\left(\frac{\hat{I}}{I_{dc}}\right)^3\sin^3\omega t+\frac{1}{24}\left(\frac{\hat{I}}{I_{dc}}\right)^4\sin^4\omega t+\ \ldots \quad (13)$$

From equations 12 and 13 have been calculated the results given in Table 2. As before it is assumed that $\hat{I}/I_{dc}$ is small enough to ensure that a negligible portion of the total second-harmonic generated arises from the presence of the $\sin^4\omega t$ term, etc. However, since the terms in equation 10 fall off in magnitude with increasing order less rapidly than in equation 13, a given high value of $\hat{I}/I_{dc}$ causes larger errors in the inverse-distortion figures of Table 1 than it does under the conditions of Table 2.

Table 2. **Theoretical output distortion percentages for pure sinusoidal input voltage to ideal junction transistor without feedback.**

Harmonic number	Distortion (%)	Distortion (%), alternative formulae for 290 K($\hat{V}_{in}$ in mV)
2	$25(\hat{I}/I_{dc})$	$\hat{V}_{in}$
3	$4.17(\hat{I}/I_{dc})^2$	$6.67\times10^{-3}\hat{V}_{in}^2$
4	$0.521(\hat{I}/I_{dc})^3$	$3.33\times10^{-4}\hat{V}_{in}^3$
5	$0.0521(\hat{I}/I_{dc})^4$	$1.33\times10^{-7}\hat{V}_{in}^4$
6	$0.00434(\hat{I}/I_{dc})^5$	$4.44\times10^{-10}\hat{V}_{in}$

Inverse distortion for parabolic device

For an amplifier having the general parabolic transfer characteristic given by equation 1 of Part 5, repeated here as equation 14, the formulae for input distortion for pure

$$v_{out}=Av'+\alpha(Av')^2 \qquad (14)$$

sinusoidal output without feedback are given in Table 3, middle column. For an ideal f.e.t. there is the restriction that the bottom of the parabola must lie on the zero-drain-current axis, as shown in Fig. 4 of Part 5, and it then follows that $\alpha\hat{V}_{out}$ may be replaced by $\tfrac{1}{4}(\hat{I}/I_{dc})$, giving the formulae in the right-hand column of Table 3. (This substitution may also be made for αV_{out} in Table 1 of Part 5 when applied to an ideal f.e.t.)

Table 3. **Theoretical input distortion for pure output for general parabolic device, and f.e.t. without feedback.**

Harmonic number	Distortion (percentage) General parabolic device	Distortion (percentage) Ideal f.e.t.
2	$50\alpha\hat{V}_{out}$	$12.5\hat{I}/I_{dc}$
3	$50\alpha^2\hat{V}_{out}^2$	$3.12(\hat{I}/I_{dc})^2$
4	$62.5\alpha^3\hat{V}_{out}^3$	$0.977(\hat{I}/I_{dc})^3$
5	$87.5\alpha^4\hat{V}_{out}^4$	$0.342(\hat{I}/I_{dc})^4$
6	$131\alpha^5\hat{V}_{out}^5$	$0.128(\hat{I}/I_{dc})^5$

Comparing the right-hand column of Table 3 with the middle column of Table 1, the input harmonics for the f.e.t., at a given $\hat{I}/I_{dc}$ are smaller and decay more rapidly with increasing order than for a voltage-driven junction transistor. However, in many practical feedback circuits, this apparent disadvantage of the junction transistor will be more than compensated by the fact that it has a much higher mutual conductance, giving a higher feedback loop gain and thus reducing all significant harmonics to a lower level than for the f.e.t.

With regard to the f.e.t. investigation of Part 5, dividing the figures determined from the right-hand column of Table 3 by 100 gives points on the left-hand vertical axis of Fig. 7 in Part 5 which coincide with the intercepts of the chain-dotted curves.

Relationship to experimental results

The distortion theorem formulated above may be used to calculate quickly and easily, the approximate output distortion for a single junction transistor stage having, say, 40dB of feedback, for $\hat{I}/I_{dc}=0.647$ as used in the experiments with the Fig. 1 circuit. The no-feedback inverse-distortion figures are determined from the middle column of Table 1, and are divided by 100 to give the predicted distortion with feedback. The values obtained are indicated by triangles on the left-hand vertical axis of Fig. 2.

As already explained, the Table 1 formulae assume $\hat{I}/I_{dc}$ is small enough for the amount of second harmonic produced by the 4th and 6th power terms in the power series to be ignored, etc. With $\hat{I}/I_{dc}$ as high as 0.647, there is, however, an appreciable error due to this cause. Calculation shows that the amounts of inverse second harmonic arising from the $\sin^4\omega t$ and $\sin^6\omega t$ terms in equation 10 are approximately 21% and 4% of the amount due to the $\sin^2\omega t$ term, the amounts produced by even higher-order terms being relatively negligible. Thus the true second-harmonic figure would be expected to be about 26% higher than that calculated from Table 1, the error becoming rapidly smaller with reduction in signal level. This more accurate theoretical prediction is indicated by the upper arrow at the left of Fig. 2, and ties up well with the broken-line extrapolation of the measured curve.

At the zero-feedback end of the Fig. 2 curves the simple theoretical distortion values are given by the middle column of Table 2, for $\hat{I}/I_{dc}=0.647$, and the values obtained are indicated by the triangles on the right-hand vertical axis of Fig. 2. As already stated, the errors under the Table 2 conditions, caused by working at a rather high signal level, are much less than for Table 1, but there are other causes of errors to be considered. Nevertheless, the calculated second-harmonic percentage agrees quite well with the experimental value. The shape of the second-harmonic curve can thus be explained in terms of the increasing effect of the high-order terms in the power series as the amount of feedback is increased – an alternative but equally sound explanation to that previously given involving intermodulation within the forward amplifier.

The theoretical zero-feedback points, marked by triangles, for harmonics higher than the second do not agree well with the measured values. The reason for this is believed to be that when the Fig 1 circuit is set for nominally zero feedback, a small but finite amount of negative feedback is effectively still in operation, mainly because of the presence of finite resistance in the base circuit. If this resistance, including $r_{bb'}$, totals 1.2kΩ, and assuming β_{ac} or h_{fe} of 500, it is equivalent to a resistance of 2.4Ω in the emitter lead,l causing $1/(1-A\beta)$ to be effectively 0.88 when set for nominally 1.0. To allow for this, the extreme right-hand plotted points on all the experimental curves should be moved to the left to $1/(1-A\beta)=0.88$. The effect of the 2.4Ω is negligible because of the logarithmic scale used in Fig. 2, except toward the right-hand side of the curves. With the curves thus shifted to the left, it seems reasonable to suppose that continuing the patterns of undulations already established, towards the $1/(1-A\beta)=1.0$ axis, would bring the curves approximately to the theoretical values marked by triangles.

When allowance is made for the production of third harmonic by the $\sin^5\omega t$, $\sin^7\omega t$ and $\sin^9\omega t$ terms in equation 10, the magnitude of the third-harmonic distortion voltage is increased by approximately 32%, raising the calculated value to that indicated by the lower arrow in Fig. 2, which again then ties up well with the broken-line extrapolation of the measured curve. The corresponding tedious calculations have not been done for the 4th, 5th and 6th harmonics, but it seems probable that they, too, would raise the levels of the points marked by triangles to give reasonable agreement with the broken-line, 45°, extrapolations of the measured curves.

There is a further small point which must now be mentioned. In Table 2, for a junction transistor without feedback driven by a sine-wave voltage, the factor $\hat{I}/I_{dc}$ appears. $\hat{I}$ is the peak value of the fundamental current and I_{dc} is the value of the collector current at the moment when the input signal voltage goes through zero. It would also be the quiescent current, if the transistor were operated at fixed bias voltage, and the mean current with the signal applied would then be larger than I_{dc} because of the rectifying action. However, the mean current is prevented from rising significantly when the signal is present in the Fig. 1 circuit, owing to the virtually-constant current in the 12kΩ emitter resistor. This results in the distortion being higher than the simple theory predicts. The fact that the measured second-harmonic curve goes through the 16% point predicted by Table 2 at its top end is thus fortuitous. The effect just mentioned tends to raise the level of the point, whereas the fact that there is a little feedback in action, even when the control is set for nominally zero feedback, tends to lower it. Once there is plenty of feedback in action both these effects become negligible.

It can thus be concluded generally that provided plenty of feedback is assumed right at the beginning, the more awkward parts of the theory outlined in this article, though academically interesting, do not need to be taken into account for design purposes. □

(To be continued)